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ALGEBRAIC STUDY ON POLYNOMIAL EQUATIONS WITH A REFERENCE TO LINEAR ALGEBRA

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ABSTRACT

In many system theories, identification and control settings, multivariate polynomial system solving and polynomial optimization problems arise as central problems. Methods for solving polynomial equations in the area of algebraic geometry have traditionally been developed. It is known as one of the most inaccessible fields of mathematics, although a large body of literature is available. In this paper , we present a method for solving polynomial equation systems using only numerical linear algebra and system theory instruments, such as realisation theory, SVD / QR, and computations of eigenvalue. By separating coefficients and monomials into a coefficient matrix multiplied by a foundation of monomials, the task at hand is translated into the realm of linear algebra. Applying the theory of realisation to the structure on the monomial basis enables all system solutions from eigenvalue computations to be found. Solving a problem of polynomial optimization is shown to be equivalent to an external problem of eigenvalue. In identification and control, relevant applications are found, such as the global optimization of structured total least-square problems.

INTRODUCTION

LINEAR EQUATIONS HISTORY -800 BC

Assyrians, Egyptians, Babylonians and Phoenicians are the most important people of the antiquity who have made considerable contributions to the subject. Of these, only the Egyptians and Babylonians have made a significant contribution to the advancement of mathematics.

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Egypt

Much of our knowledge of ancient Egyptian mathematics comes from two Papyri, namely the Rhine Mathematical Papyrus and the Moscow Mathematical Papyrus, which contain a collection of mathematical problems with their solutions. The Rhine Mathematical Papyrus, named after the Scottish man, A. H. Rhind, who bought it at Luxor in 1858. Moscow Mathematical Papyprus, bought by V.S. in 1893. Golenishchev was later sold to the Moscow Fine Arts Museum. The former papyrus was copied around 1650 by the scribe Ahmes from an original dating back to 200 years. The latter papyrus is approximately of the same period. The Rhine Papyrus has 85 problems and the Moscow Papyrus has 25 problems. In the first part of the papyrus[24], Ahmes deals with fractions of a unit (in modern) terms) it will be ofthe formAhmes deals with the fractions of the unit, (in modern terms) they will be of the form. $2/2n-1$ where n stands for all odd numbers from 5 to 49.

For example, 2/29 can be written a $\frac{1}{24}+\frac{1}{58}+\frac{1}{174}+\frac{1}{132}$ See how close to the scribe was 24 58 174 132? Ahmes only records the result. It is possible that they have been worked out by former mathematicians, each of whom has worked in his own way or through repeated trials. It's hard to know how these results have been achieved.

Most of the mathematical works of the ancient times are concerned with the solution of problems to which various mathematical techniques are applied. Our study of these problems begins with several methods of solving linear equations. Many of the works take the solution of single linear equations for granted. Whenever such an equation appears to be part of more complex problems, the answer is merely given without any method of solution being mentioned. But these equations are explicitly dealt with by the Egyptian papyri.

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The Moscow Papyrus, for example , uses the usual current technique. It shows the number such that if it is taken $1\frac{1}{2}$ t times and then 4is. added, the sum is 10 [36]. In modem terms, the equation is $1\frac{1}{2}x+4=10$. The two scribes are proceeding as we do today. First, he subtracts 4 out of 10 to get 6. Then multiplies 6 by 2/3 (reciprocal) $1\frac{1}{2}$) to get 4 as a solution.

The problem of Rhind Papyrus in [36] asks that the sum of itself, its 2/3, its 1/2 and its 1/7 be 33. That's, in modem terms, to find 'x' $x+\frac{2}{3}x+\frac{1}{2}x+\frac{1}{7}x=33$

The scribe resolved the problems by dividing the 33 by $1+\frac{2}{3}+\frac{1}{2}+\frac{1}{7}$.

His answer in modern notation is $14\frac{28}{97}$

These two problems are presented as purely abstract, with no reference to the quantity of real life, The Scribe has shown that his technique works for any problem of division.

NON-LINEAR POLYNOMIAL SYSTEMS

Current known techniques for the resolution of non-linear polynomial systems can be classified into symbolic, numerical and geometric methods. Symbolic methods based on the resultant and Gr}obner bases algorithms can be used to eliminate variables and thus reduce the problem to the end of univariate polynomial roots. These methods are rooted in algebraic geometry. However, current algorithms and implementations are ancient for low-grade polynomial systems consisting of up to three to four polynomials only. The ma jor problem arises from the fact that the computing roots of univariate polynomials may be ill-adjusted for polynomials of a degree greater than 14 or 15, as shown by Wilkinson[8]. As a result , it is difficult to implement these algebraic methods using night precision arithmetic and slows down the resulting algorithm. As far as the use of algebraic methods in geometric and solid modelling is concerned, the current view is that they have led to a better theoretical understanding of the problems, but their practical impact is not clear [7, 9].

The numerical methods for solving polynomial equations can be classified into iterative and homotope methods. Iterative techniques, like Newton's method, are only good for local analysis and work well if we give each solution a good initial guess. This is rather difficult for applications such as intersections or geometric constraint systems. Homotopy methods based on follow-up techniques have a good theoretical background and follow paths in a complex space. In theory, each path converges towards a geometrically isolated solution. They have been implemented and applied to a wide range of applications[10]. In practise, there are many problems with the current implementations. The De

The erent paths being followed may not be geometrically isolated, causing problems with the robustness of the approach. In addition, continuous methods are considered to be computationally very demanding and currently limited to the solution of dense polynomial systems only. Recently, methods based on arithmetic interval have received a great deal of attention in computer graphics and geometric modelling. The resulting algorithms are robust, although their convergence may be relatively slow.

For some specific applications, algorithms have been developed using the geometric formulation of the problem. This includes subdivision-based algorithms for curves and surface intersection, ray tracing. In general, subdivision algorithms have limited applications and their convergence is slow. The Bezier Clipping[11] improved their convergence. However, algebraic methods have been found to be the fastest in practise for low degree curve intersections. Similarly, algorithms based on the geometric properties of the mechanisms have been developed to address the problems of kinematics, constraint systems and motion planning.

Numerical Linear Algebra.

The second half of the 20th century witnessed the maturation of the numerical linear algebra, which has become a well-established field of research. A multitude of reliable linear algebra numerical tools[20] is well understood and developed. We will outline an independent value-based solution method in which linear algebra notions such as row and column space, zero space, linear (in)dependence and matrix rank play an important role, as well as essential linear algebra-based numerical tools such as singular value decomposition and self-value decomposition. The central object of the proposed method is the Macaulay matrix [35, 36], a sylvester-like structured matrix based on the coefficients of the multivariate polynomial set.

The Macaulay matrix is obtained by considering multiplications (shifts) of the monomial equations so that the result has a certain maximum degree. Translates a system of polynomial equations into a system of homogeneous linear equations: the polynomials are represented as (rows of) the Macaulay matrix multiplied by a multivariate Vandermonde vector containing the monomials. The proposed method proceeds by iterating by increasing the degree to which the Macaulay matrix is built. We will study the dimensions, rank and (co-)rank1 of the Macaulay matrix to an increasing degree. The Macaulay matrix will be over-determined to a certain degree. Interpreting this as a homogeneous system allows us to divide the unknowns (monomials) into linearly independent and linearly dependent unknowns. We will see that the corank of the Macaulay matrix corresponds to the number of linearly independent monomials and, furthermore, is equal to the number of solutions.

REVIEW OF LITERATURE

The first mathematician was a Greek man named Zeno. Zeno is memorable for the proof of three theorems: I that motion is impossible; (ii) that Achilles can never catch a tortoise (he did not notice that this is the result of his first theorem); and (iii) that half the time may be equal to twice the time. The other Greeks did not consider this a very good start, so they turned their attention to geometry.

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Pythagoras (569-500 B.C.) was born on the island of Samos, Greece. Legend has it that Pythagoras sacrificed 100 oxen after the completion of his famous theorem. Although the discovery of the famous theorem is credited to him, it is impossible to tell whether Pythagoras is the actual author. The Pythagoras had discovered irrational numbers. If we take the right isosceles triangle with the legs of measure 1, the hypotenuse will measure. But this number can not be expressed as a length that can be measured by a ruler divided into fractional parts and deeply disturbed by the Pythagoreans, who believed that "All is number." They called these numbers "alogon," which means "unspeakable." So shocked by these numbers, the Pythagoreans killed a member who dared to mention their existence to the public. It would be 200 years later for the Greek mathematician Eudoxus to develop a way of dealing with these unspeakable numbers.

Pythagorean Triple: A Pythagorean triple (like 3-4-5) is a set of three whole numbers that work in the Pythagorean theorem and can therefore be used for the three sides of the right triangle. Euclid, 300 BC, invented geometry. The 13 books of the elements describe the geometrical facts of the triangle circles and other planar and spatial figures. This is the first axiomatic exposure of mathematics (now the only approach used).

Parallel postulate: Given the line L and the point P not on the line L, then there is exactly one line through P that does not meet the line L. Euclid felt uncomfortable using this axiom and tried to prove it as much as possible without using it.

Archimedes (287 BC to 212 BC) is very memorable for bathing. Unfortunately , despite his principles, he forgot to get dressed afterwards. Archimedes was the first to use the Δ symbol and gave the approximation: $3 + \frac{10}{17} < \pi < 3 + \frac{1}{7}$.

Archimedes invented a method of fluxion to calculate the volume of certain solids. This is closely linked to integration, as we know it, and contains the seminal idea of limitation.

Diophantus (250 BC) was the first to examine the integral and rational solutions of type equations.

$x^n + y^n = z^n$, Last Fermat's Theorem

$x^2 - Ny^2 = \pm 1$, Pell's equation

:

From that time on there was an open interval, the other end of which was Descartes (1596 to 1650), who was divinely inspired to invent analytic geometry, and was once found sitting inside a stove to keep himself warm. He also discovered that he existed, and he was able to prove it.

Newton (1643to 1727) was indeed very memorable, mainly because he had just missed living in St. John's. He invented the Calculus to console himself. Newton is also famous for being admired by Taylor, who invented Maclaurin 's series and admired Newton. Taylor, however, lived in St. John's, so he was luckyr than Newton.

The next major mathematician is the Bernoullis. Despite having invented numbers, no one knows how many of them there were, and he's been living all over the century. It was called Nicholas, Jacob and John, and one of them was called Daniel.

Euler (1707 to 1783), Langrange (1736 to 1813) and Laplace (1749 to 1827) were all famous for inventing equations. Only one of Laplace 's equations is well known, but that's enough for anyone. It makes electricity and hydrodynamics much easier for people who don't have to solve the problem. Euler and Lagrange both dealt with a number of things, which caused the calculation of variations. That was both memorable and regrettable.

Gauss has invented so many things that it's just not true. These included Earth's magnetism, equation theory, Cauchy 's theorem, and Cauchy-Riemann 's equations. In fact, whenever anyone invented anything in the first half of the 19th century, Gauss had invented it twenty years earlier, and was still alive to tell him that. He was born in 1777, died in 1855, and spent many years in between. He was a very memorable man, and a good thing.

Cauchy 's theorem is very important, but it's much harder to prove now than it was when Gauss invented it.

Lobatchewski (1793 to 1856) had failed a geometry examination when he was at school, for he made things harder for everyone by inventing non-Euclidean geometry-just to get his revenge, of course. This was particularly bad for the railroads, as it made parallel lines so much more difficult.

Hamilton (1805-1865) was an Irishman. When he learned 13 languages before he left school, he decided that there was no future in this, and he looked at mathematics. He invented the Hamilton Principle, the Hamiltonian, the Hamilton Jacobi Theorem, and the Hamilton-Cayley Theorem, but not the Hamilton Academics. He also invented quaternions towards the end of his life, but no one except himself ever fell in love with them.

Weierstrass (1815 to 1897) is memorable because of Sonja Kowalewski (1850 to 1891), who is, of course, memorable because of Weierstrass. He said that if you put an infinite number of things in a small space, some of them would be pretty close together.

John Couch Adams (1819 to 1890) was the most memorable of all mathematicians. He had the good fortune to live in St. John's, and he was named after the company. He discovered Neptune just after Leverrier, and he would have discovered it before if the Royal Astronomer had kept his eyes open.

Charles Lutwidge Dodgson(2013) was a minor Oxford mathematician who must not be confused with Lewis Carroll, whom he impersonated when he sent copies of his work to Queen Victoria. They lived at the same time.

Carroll's main problem was that of the Cheshire cat. His treatment is essentially unsound, however, as he says, "This time the cat vanished quite slowly, beginning with the end of the tail, and ending with the grin, which remained some time after the rest of it had gone." It is evident that, by the time the tail had disappeared, the cat would have been a Manx cat. This is a contradiction, because it was a Cheshire cat, by hypothesis. Carroll also discussed the increased angular velocity of the world if everyone had their own business in mind.

Riemann (1826 to 1866) invented the tensor calculus, which led to the theory of relativity.

Algebra may be divided into "classical algebra" (equation solving or "finding unknown number" problems) and "abstract algebra," also known as "modern algebra" (study of groups, rings and fields). Classical algebra has been developed over 4000 years. Abstract algebra has only appeared in the last two hundred years. As algebra grows out of arithmetic, the recognition of new numbers — irrational, zero, negative, and complex numbers — is an important part of its history.

India has a glorious past in mathematics , especially in algebra. There is no doubt that there was a time when great scholars worshipped, and those eager to learn would flock around for knowledge. It is therefore interesting to know the names and activities of these scholars who are pioneers in the field of algebra.

The meaning of the equation is the formula affirming the equivalence of the two expressions connected to the sign. Equations (Samikaran) appear to have been classified in Sthanaga-Sutra (c. 300 B. C.) according to the powers of unknown quantity, e.g. yavat (simple), varga (quadratic), ghana (cubic), varga-varga (biquadratic), etc. But this classification has not been maintained.

Brahmagupta (628 A. D.) has given the following classifications: I eka-vargasamikararna-equations in one unknown, consisting of linear and quadratic equations; (ii) aneka-vargasamikararna-equations in many unknowns; (iii) bhavita-equations involving unknown products.

This classification was further elaborated by *Prthudakasvami (860 A.D.) and Bhaskara-II (1150 A.D.).* This primitive method of solving simple linear equations of type 0, by substituting the guess values a1, a2 etc. for widespread use among middle-aged Arab and European mathematicians.

OBJECTIVES OF THE STUDY

1. To study the theory of solutions and the structure of solutions for linear algebra equations.
2. The study of polynomial equations appears in a number of mathematical models.

CONCLUSION

This paper summarises the study entitled "History of Mathematics: An Algebraic Study of Polynomial Equations with Special Reference in Linear and Quadratic Equations." An attempt has been made in this study to shed light on the growth of algebra, particularly linear and quadratic equations. Ancient polynomial equations, mediaeval polynomial equations, early modern and modern polynomial equations and algebraic concepts of Egyptians, Babylonians, Indians, Greeks, Chinese, Arabs, Europeans, etc. examined and presented significant aspects in order to portray the historical basis of algebra spanning over a period of more than three thousand years (1650 BC-Modem period [from 1701 AD to da da]. The evolution of ideas and a number of concepts pertaining to algebra can be discerned from the accounts collected, compiled and interpreted from secondary sources.

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